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PEDAGOGICAL CONDITIONS FOR FORMING BILINGUAL MATHEMATICAL COMPETENCE IN BASIC SCHOOL STUDENTS*

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Introduction. In the process of bilingual education, schoolchildren must not only qualitatively master the content of the subject but also overcome language difficulties. There is a connection between speech and mathematical activities. The essence and structure of bilingual mathematical competence are based on this relationship, allowing bilingual students to effectively acquire knowledge in the conditions of national-Russian bilingualism. We have also proposed ways of forming bilingual mathematical competence focused on developing mathematical speech culture and teaching schoolchildren to use multicultural knowledge.

Aim. The article aims to characterize the pedagogical conditions directed at the emergence of bilingual mathematical competence among basic school students (grades 5 to 9) within national-Russian bilingualism.

Material and methods. The study relies on theoretical methods of comparative analysis, synthesis, and generalization provided by the scientific and methodological literature on the researched topic.

Results and discussion. Works indicating a clear relationship between the language of instruction and the subject of Mathematics were analyzed. The need to take into account the mother tongue of schoolchildren in bilingual education was established. In addition, it was found that the degree of native and Russian language proficiency affects the mathematics achievement of bilingual students. According to the analysis, bilingual education should lead to the emergence of competencies distinguished by a high level of language proficiency and high-quality mastering of the subject.

Conclusion. The concept of "bilingual mathematical competence" got a detailed description in the course of the research. This concept combines components of a school subject, languages (native and Russian), and a component of intercultural communication. The following pedagogical components were described:

1) tasks aimed at mastering terminology, symbols, and graphic images; verbal and logical constructions of the mathematical language; written educational texts;

2) illustrated Yakut-Russian, Russian-Yakut terminological dictionary in mathematics for the 5th and 6th grades, which includes 349 terms and set phrases;

3) bilingual strategies aimed at reducing the linguistic complexity of mathematical problems (by replacing unfamiliar or rare words; changing the passive voice to active verb forms; reducing long names and indications; highlighting individual conditional sentences, or changing the order of the conditional and main sentences; replacing complex questions to simple ones; clarification of abstractions using more specific information);

4) methods and techniques of bilingual teaching of mathematics (consecutive translation, visual aids, immersion teaching, semantization);

5) tasks that contain historical, ethnocultural, and local history materials.

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Introduction

Recently, the development of bilingual education has become a growing trend all over the world. Various options of its implementation are used: 1) based on the languages spoken by a linguistic majority/minority; 2) based on the official language of the state, as well as the languages of ethnic groups; 3) based on the native and foreign languages [1, p. 91]. Citizens of the Russian Federation have the right to get preschool, primary and basic education both in their native language and in Russian [2]. Russia is a multinational state, and there are 277 languages and dialects, 30 of which are used as the language of instruction [3]. The Republic of Sakha (Yakutia) has officially adopted the second state language – the Yakut language (Sakha) [4], which, along with Russian, is the language of instruction. In Yakutia, from 1917 to the present, the following models of bilingual education have been formed: the "Linguistic Heritage" program, a transitional model, and immersion education [5]. According to the experience of basic education organizations that implement the native (Sakha) language of instruction in primary grades, bilingualism (the process of alternating use of languages [6, p. 22]) is formed with an emphasis on the native language of students. In the 5th and 6th grades of the middle school, there is a gradual transition from the native language to Russian; in the 7th to 11th grades, on the contrary, bilingualism with an emphasis on the Russian language is observed.

In the context of the Russian national bilingualism, in which the first component of bilingualism is the native language, and the second is Russian [7], a study of Mathematics is often associated with mathematical and linguistic difficulties. According to M. K. Cirillo, R. Bruna, B. Herbel-Eisenmann [8], and P. Ron [9], it would be a mistake to believe that even students with a high level of language proficiency can automatically master the oral and written forms of mathematical speech. It is evident that in national schools, language difficulties may be more pronounced when teaching mathematics. We believe that the poor level of Russian language proficiency and the flow of thought processes mainly in their native language can cause these difficulties.

Since studying mathematics, like any other academic discipline, is impossible without mathematical and natural languages [10, 11], the relationship between speech and mathematical activity should be considered in educational practice. Thus, this article clarifies the concept and structure of bilingual mathematical competence, which allows students to successfully master the primary school curriculum in the conditions of national-Russian bilingualism; Presented below are ways of forming such competence.

Materials and methods

Within the framework of this study, domestic and foreign scientific and methodological literature was analyzed. The synthesis and generalization of the data obtained during the analysis made it possible to reveal the meaning of the concept "bilingual mathematical competence" and describe the forms, resources, and methods of its formation in the conditions of national-Russian bilingualism.

Let us consider the relationship between the language of instruction and mathematical content. The results of many foreign studies show that the mathematical and language skills of students are closely interrelated [12]. Several studies indicate that language skills [13], reading comprehension [14], and vocabulary [15] can be identified as significant predictors of the development of math skills. K. Bochnik and S. Ufer [16] proved that subject-specific language skills partially mediate the relationship between general language and math skills. In their study, S. Prediger and L. Wessel noted the significant role of subject-specific language registers necessary for understanding the meaning of mathematical concepts [17]. By "register," we mean a functional variety of a language in various situational contexts (a text consisting of lexical and grammatical units typical for a particular communicational situation) [18]. According to M.A.K. Halliday, the term "mathematical register" denotes language expression for mathematical purposes, where natural languages play a significant role in the expression of mathematical ideas. Just like other natural languages, a mathematical language has some specific features [19]. It is known that a mathematical language is used to describe representations, examples, or phenomena associated with previously studied mathematical concepts. It includes the vocabulary specific to the subject and more complex skills such as the derivation of mathematical structures described verbally [20].

Let us highlight some studies that have identified the differences between casual and academic language registers [21]. S. Prediger and L. Wessel believe that mathematical concepts within classroom discourse are described according to a specific register [17]. The school language register, which is part of the academic language register [22], is located between the informal register and the technical register, which describes language in teaching mathematics as a school subject [17]. Members of the National Council of Teachers of Mathematics also believe that there is a "more mathematically structured" language between casual and academic languages [23]. Let's consider that the actual mathematical language is an extension of the natural language [24], then the use of the casual spoken language can be viewed as the basis for developing the mathematical language.

Many scientists believe that academic achievements are associated with general language competence and text comprehension [25–29]. The reasons for this underlie the educational and linguistic requirements of the subject "Mathematics" (for example, reading and understanding the texts on mathematical problems) [30]. Since the language carries two functions (communicative, cognitive), it is difficult for learners to overcome the language requirements in the oral and written environment when teaching mathematics [31, 32]. It is evident that the conditions of national-Russian bilingualism exacerbate this problem. According to L. Wessel, the use of the native language in multilingual classes (especially at the initial stage) is crucial for forming and using an abstract mathematical language in speech [33]. Many studies on multilingualism in the educational environment show how important it is to take into account the native language of students when using a second language as the language of instruction [34]. Indeed, bilingual students who speak both languages at a sufficiently high level of proficiency show excellent results in math education [23, 35–39]. A smooth transition of instruction language from the native language to Russian helps schoolchildren overcome linguistic and subject difficulties in teaching mathematics [40].

Following L. T. Zembatova, we understand the concept of "bilingual teaching in mathematics" "as an interconnected activity of a teacher and a student, aimed at the formation of mathematical knowledge using the native and Russian languages resulting in the deep, conscious acquisition of mathematical content, the development of mathematical speech, the formation of a culture of mathematical thinking, as well as in increasing of proficiency level in a second language (Russian)" [41, p. 177].

The result of bilingual education is the synthesis of specific competencies, ensuring a high level of language proficiency and deep mastery of subject content [42, 43]. According to The Threshold Theory, a necessary condition for achieving a positive influence of bilingualism on the intellectual development of schoolchildren is the formation of bilingual competence. J. Cummins [23, 44] distinguishes two levels of bilingual competence: 1) "BICS" (Basic Interpersonal Communicative Skills) – basic language proficiency at the level of everyday communication;

2) "CALP" (Cognitive Academic Language Proficiency) – the use of a second language at a higher level in the learning process.

To reveal the essence of the "bilingual mathematical competence" concept, let us analyze concepts that are close in meaning to it. A.V. Khutorskoy defines the concept of "competence" as a set of interrelated personal traits (knowledge, abilities, skills, methods of activity) related to a specific range of objects and processes which are necessary for achieving productive activity in interaction with these objects and processes [45].

N. Chomsky [46] defines the concept of "linguistic competence" as the ability to understand and reproduce an unlimited number of correct sentences through the acquired linguistic signs and the rules for their connection. He also believes that linguistic competence is perfect grammatical knowledge, which is always correlated with knowledge of a language system.

D.H. Hymes [47] expanded the concept of "language competence" and introduced the concept of "communicative competence," which denotes the sum of language skills and knowledge of the speaker/listener under changing situations and conditions of speech.

Yu.L. Semenova studied the formation of bilingual communicative competence of schoolchildren and defined it as "the ability (mastery of subject and language competences in two languages) and students' readiness (competence of personal self-improvement) to carry out effective interpersonal, intergroup and intercultural communication both in their native language and foreign language" [48, p. 69].

Some scientists [45, 49–51] believe that the concept of "subject competence" includes the abilities required to perform specific actions in any subject category and narrow-subject knowledge, skills, and abilities as well as methods of thinking. In particular, mathematical competence is the ability to structure data (a situation), isolate mathematical relations, create a mathematical model of a situation, analyze and transform it, and interpret the results obtained [52].

So, to define the concept of "bilingual mathematical competence," we will operate with such concepts as "knowledge," "skills," "ability," and "readiness" [53].

Abilities are individual psychological characteristics of a personality expressed in its activities that are conditional for the success of the activities. The overall mastery of knowledge, skills, and abilities (in terms of depth, easy-learning, high learning pace) depends on abilities, but they are not limited to knowledge and skills [54].

Readiness is also based on the activity approach and implies one's desire to do something. In pedagogy, "readiness" is used as an integrative concept and includes ideas about readiness for certain activity types, such as readiness for school teaching [55, p. 148].

Theoretical analysis of the literature showed that in modern pedagogy, despite extensive data on the competence-based approach in education, the problem concerning the formation of subject competence in the process of bilingual teaching of mathematics is not given due attention. Among the researches, we would like to note the works related to the formation of bilingual subject competence in mathematics for primary school students [56] and higher educational institutions [43]. Based on the definitions by L. L. Salekhova [43] and L.T. Zembatova [56], we define bilingual mathematical competence of primary school students as a didactic category denoting a set of intercultural and special mathematical knowledge, skills, and abilities that ensure the readiness to implement successful educational activities in the native and Russian languages in the conditions of national-Russian bilingualism. We also clarify its structural composition, which consists of the following components: subject (mathematics), special language (native language), special language (Russian), and intercultural component.

The mastery of the school curriculum in mathematics and the level of mathematical thinking among students is reflected in the subject component of bilingual mathematical competence. The subject component consists of knowledge system of the scientific conceptual mathematical apparatus (basic laws of mathematics, mathematical concepts), mathematical language (semantics and syntax), universal mathematical methods (mathematical description of processes, mathematical modeling), as well as skills and abilities of mathematization of empirical material (application of the concepts and methods of mathematics for the quantitative analysis of processes and phenomena of the world), the logical organization of mathematical material, and the application of mathematical theory (the ability to apply mathematical concepts, mathematical methods and mathematical language, extract mathematical information from educational texts, translate the information received into the language of mathematics, solve mathematical problems, perform computational actions, use computer technologies, evaluate mathematical objects and phenomena from the position of previously acquired knowledge, present mathematical objects in the form of diagrams, graphs, formulas).

The language components in the native and Russian languages consist of general language and speech competencies and include students' mathematical speech in their native and Russian languages. These components also characterize the degree of language proficiency of schoolchildren and their ability to use languages in speech. A sufficient level of language components allows students to use mathematical language based on their native and Russian languages, such as explaining the material covered, describing objects or conditions, introducing mathematical concepts, commenting on the problem-solving situations.

A sufficient formation level of the intercultural component allows bilingual schoolchildren to apply multicultural knowledge in bilingual education, allowing them to use more methods of mental activity, thereby deepening and consolidating the knowledge gained and also making it easier to participate in communication with members of a multicultural society.

Results and discussion

Let us describe the methodology for the formation of bilingual mathematical competence. In order to implement successful educational activities in the native and Russian languages, the following principles can be applied, taking into account the linguistic properties as a means of teaching [57]:

1. Integrated language and subject learning (using the native language of learners, observing and providing support to learners, understanding subject matter, and supporting learning processes through task-oriented language work).
2. There is speech attention and speech consciousness (specific and consciously developed speech action, awareness, and reflection of linguistic phenomena, terms, or structures).
3. Active actions and interaction of languages (stimulating students to participate in active speech activity).
4. Transparency of language requirements (clarification of language learning goals along with subject goals).
5. Systematic language support (teacher assistance only if necessary, when the student cannot cope with the task independently).
6. Emphasis on written speech (stimulating lengthy consistent oral and written texts).
7. Emphasis on working with text (providing a plan for writing and reading, operating with longer texts).

Applying these principles helps ease the language difficulties that bilingual children experience in the teaching of mathematics. In order to follow these principles, it is necessary to use bilingual teaching methods. Scientists have different opinions on the methods of bilingual education. Based on the works of A.G. Shirin [42], N. Masch [58], M.N. Pevzner [59], E. Turman

[60], E.S. Pavlov [61], it is possible to distinguish a set of methods of bilingual education: 1) methods of teaching mathematics; 2) methods of teaching native and Russian languages; 3) general didactic methods: traditional (frontal, teacher's report, standardized conversation, reproductive-response method); developing methods (work in group and pairs, discussion, debate, role play, panel discussion, brainstorming, problem-based learning); open methods (free activity, project activity, independent activity, individual educational project, information technology); 4) special methods and techniques: immersion methods (total and soft immersion); language support (visual support, reading support, language support); bilingual teaching techniques (input, bridging prompting, code-switching).

These teaching methods are also applicable for the formation of bilingual mathematical competence. In addition to textbooks, we suggest using a system of mathematical tasks aimed at developing mathematical speech in schoolchildren (as in the case of the Yakut-Russian bilingualism). The system forms the subject and special language components (native and Russian) of bilingual mathematical competence. Tasks are presented in parallel texts in the native (Yakut) and Russian languages, i.e., texts in one language and their translation into another language [62]. The task system consists of the following components:

1. Tasks designed for working with terminology, symbols, and graphical images:

– explanation of terms, symbols, and symbolic expressions, the origin of terms; correlation of terms with each other; explanation of the symbols meaning and symbolic expressions;

– transition from a graphical form of notation to a verbal-symbolic form ("reading" of graphical images);

– transition from a symbolic (verbal) form of notation to a graphical presentation;

– writing mathematical sentences (or individual terms) using symbols;

– reading symbols;

– transformation of symbols;

– terminological vocabulary test;

– consecutive translation.

2. Tasks designed to work with the verbal and logical constructions of the mathematical language:

– finding false or missing features in the definitions of mathematical concepts;

– finding errors in the definitions of mathematical concepts;

– true or false statements;

– students' independent wording of mathematical sentences.

3. Tasks designed to work with written training texts:

– finding unknown words, language phrases, and symbols in the text;

– finding errors in the text;

– making a coherent text from "scattered" sentences (or fragments);

– filling in gaps in the text.

4. Tasks designed for working with text tasks (commenting on solving a text problem).

For example, let us consider tasks requiring students to explain the meaning of terms and symbolic expressions.

Table 1

Math problems in the native language (Sakha) and the Russian language requiring an explanation of the term

1 холобур. «Сөптөөх доруоп» тиэрмин суолтатын тылгынан быһаар (быһааран суруй).	*Example 1.* Объясни значение термина «правильная дробь». (Explain the meaning of the term «correct fraction»)

The answer in the native language of the students can be as follows: «Знаменателэ числителинээҕэр улахан көннөрү доруоп сөптөөх доруоп буолар. Холобур, знаменателгэ турар 2 чыыһыла числителгэ турар 1 чыыһылатааҕар улахан буолан, икки гыммыт биирэ доруоп сөптөөх доруоп буолар. 2 > 1, 1/2 – сөптөөх доруоп».

The answer in Russian can be as follows: «Правильная дробь – это обыкновенная дробь, в которой числитель меньше знаменателя. Например, дробь одна вторая является правильной дробью, так как в числителе стоит натуральное число 1, которое меньше числа 2, стоящего в знаменателе дроби. правильная дробь, так как 1 < 2». (A regular fraction is an ordinary fraction in which the numerator is less than the denominator. For example, a one-half fraction is a regular fraction, since the numerator contains a natural number 1, which is less than the number 2 in the denominator of the fraction. It is a regular fraction since 1 < 2)

Table 2

Parallel text translations of a math problem in the native (Sakha) and Russian languages, requiring an explanation of the meaning of symbolic expressions

2 холобур. Бэриллибитхолобурдар-тансөптөөх холобуру бул: a) НОД (4;18) = 18; b) $\|-3\frac{1}{4}\|<1\frac{1}{2}$; c) 34 $\cdot(-7)=-238$. Эппиэккин быһаар	*Example 2.* Какие из символических записей правильные? Объясни свой ответ: a) НОД (4;18) = 18; b) $\|-3\frac{1}{4}\|<1\frac{1}{2}$; c) 34 $(-7)=-238$ (Which of the symbolic notations are correct? Explain your answer: a) GCD (4; 18) = 18; b) $\|-3\frac{1}{4}\|<1\frac{1}{2}$; c) 34 $(-7)=-238$

The answer in the student's native language: «Сөптөөх холобур ууруктаах уонна мэлдьэхтээх чыыһылалар төгүллээһиннэрэ буолар (34 • –7 = –38. Ууруктаах уонна мэлдьэхтээх чыыһылалары төгүлллүүргэ бу чыыһылалар муодулларын төгүллээн этиллии суолтатын булабыт. Ууруктаах уонна мэлдьэхтээх чыыһылалар үөскэмнэрэ мэлдьэхтээх чыыһыла буоларын иһин, тахсыбыт чыыһыла иннигэр «–» бэлиэни туруорабыт. НОД (4;18) = 18 холобур суолтата суох буолар. 4 чыыһыла 18 чыыһылаҕа түҥэтиллибэт буолан, 4 уонна 18 чыыһылалар саамай улахан уопсай түҥэтээччилэрэ 18-ка тэҥнэспэт. Ханнык баҕарар чыыһыла муодула ууруктаах чыыһыла буолан, икки ууруктаах булкаас чыыһылалары тэҥниибит. Бэриллэбит мэлдьэхтээх чыыһыла муодула ууруктаах чыыһылатааҕар улахан буолан, тэҥэ-суох сыыһа холобур буолар».

The answer in Russian: «Запись НОД (4; 18) = 18 неверна, так как число 18 не является делителем числа 4, поэтому наибольший общий делитель чисел 4 и 18 не может быть равен 18. Также не имеет смысл запись , так как модуль отрицательного числа – 3 1/4 равен 3 1/4. Сравнив два смешанных положительных числа, выясним, что 3 1/4 больше 1 1/2. Среди данных примеров правильным оказался пример, где предтавлена запись 34 ·(– 7) = –238, так как при умножении чисел с разными знаками модули этих чисел перемножаются, а перед произведением ставится знак «–». В результате умножения положительного числа 34 и отрицательного числа (–7) получаем отрицательное число (–238)» (Recording GCD (4; 18) = 18 is incorrect, since the number 18 is not a divisor of the number 4, therefore the greatest common divisor of the numbers 4 and 18 cannot be equal to 18. It also makes no sense to write, since the modulus of a negative number is – 3 1/4 is equal to 3 1/4. Comparing the two mixed positive numbers, we find that 3 1/4 is greater than 1 1/2. Among these examples, the correct example turned out to be where the notation 34 · (– 7) = – 238 is presented, since when multiplying numbers with different signs, the moduli of these numbers are multiplied, and a "–" sign is placed in front of the product. As a result of multiplying a positive number 34 and a negative number (–7), we get a negative number (– 238)).

The answers of bilingual students must be accurate and proper, i.e., mathematical terms and expressions correctly should be written correctly (following the literary native and Russian languages), sentences must be formulated precisely, their explanation must be complete, notes must be made accurately. In addition, the reasoning of children must be logically structured so that they can come to the correct conclusion. In other words, the communicative qualities of mathematical speech among schoolchildren should be at a sufficiently high level. For example, a teacher can periodically monitor the development of the basic communicative qualities of mathematical speech (correctness, consistency, accuracy, relevance) [63], the level of formation of which shows the level of development of the culture of mathematical speech as a whole. Students should consciously switch from one language to another when providing an answer while not mixing them. A solution can also be presented orally.

Such tasks allow us to apply the above principles in Maths class and use the techniques and methods of bilingual learning to control the processes of switching and mixing language codes and avoid the negative consequences of language contacts and interferences.

It is necessary to offer students word problems containing the following materials to form an intercultural component in Maths lessons: historical (historical events, biographies of mathematicians), ethnocultural (traditions, culture, national values, experience-based knowledge of peoples), as well as materials based on local history (geographical, cultural, historical, economic, ethnographic features of Russia and the republic).

In addition to the tasks system in Maths lessons, a dictionary can be used as an additional teaching aid, for example, a dictionary of mathematical terms [64], visual support cards, comparison tables, and Internet resources.

Conclusion

Since the study of mathematics is closely related to language processes, the interdependence between speech and mathematical activity should be taken into account in educational practice. In the conditions of national-Russian bilingualism in schools, bilingual education should be focused on developing competencies in schoolchildren, ensuring the achievement of a high proficiency level of mathematical speech in two languages, and the ability to communicate with members of a multicultural society. That is, the result of bilingual teaching in mathematics should be considered the formation of bilingual mathematical competence.

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